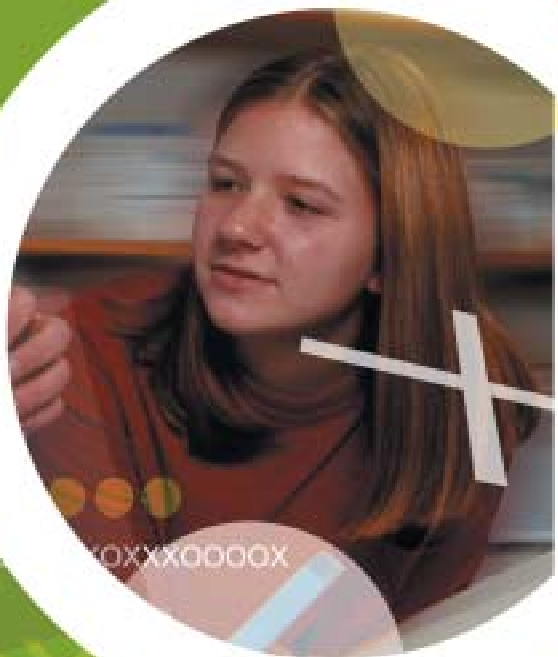
Department of Education and Training

MONITORING STANDARDS IN EDUCATION

Student Achievement in Mathematics:

Western Australian Government Schools 2002



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The 2002 Monitoring Standards in Education report on students' achievements in mathematics represents a collaborative effort by many Western Australian teachers, educators and researchers.

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Introduction

MONITORING STANDARDS IN EDUCATION TESTING PROGRAMS

For over a decade, Monitoring Standards in Education (MSE) has been reporting on student achievement in Mathematics in Western Australian government schools.

In 2002, testing was undertaken in the Mathematics learning area for the sixth time, following assessments in 1990, 1992, 1996, 1998 and 2000.

The assessment tasks developed for the 2002 program measured, for the second time, Years 3 and 7 students' achievement in Working Mathematically and Year 10 students' achievement in all six strands of the Mathematics learning area.

THE VALUE OF THE MSE TESTING PROGRAMS

Monitoring Standards in Education testing programs report student performance and the relationship between achievement and certain background variables, as well as generating comparisons between sub-groups of students. The assessments are part of the Department of Education and Training's public accountability procedures and the results provide a sound basis for initiating developments that will further improve the outcomes of education.

MSE produces a range of assessment materials for school use. The tests give a school the capacity to observe, validly and reliably, trends in student performance over time and to describe how the school compares with the rest of the school system.

REPORTING FRAMEWORK

The MSE testing programs focus on reporting student achievement in relation to the *Curriculum Framework* (1998). The level of student achievement is aligned with the eight levels of achievement described in *Outcomes and Standards Framework* (OSF). This is achieved by using the Rasch measurement model to map student achievement onto a scale that links the range of skills observed during the assessment with the eight achievement levels.

Summary of 2002 Results

BACKGROUND

The aim of the MSE 2002 Mathematics Testing Program was to collect information on skills, understandings, problem-solving strategies and Working Mathematically processes of a representative sample of Years 3, 7 and 10 students in Western Australian government schools.

Years 3 and 7 students were tested in Working Mathematically only.

Year 10 students were tested across all the six strands of the *Outcomes and Standards Framework:* Working Mathematically, Space, Measurement, Chance and Data, Number, and Algebra.

The analysis of MSE 2002 data using the Rasch measurement provided strong evidence that Working Mathematically and Content test items belong to the same measurement scale. The construction of a single scale that gives interval measures of student achievement on all the six mathematics strands was one of the main achievements of the MSE 2002 program.

YEAR GROUP PERFORMANCES

YEAR 3

The mean performance of the Year 3 group was within the boundaries of level 2.

About 80 per cent of the Year 3 students displayed Working Mathematically processes and understandings associated with at least level 2 outcomes and about 40 per cent with at least level 3 outcomes.

There was a decline in mean Year 3 performance from 2000 to 2002.

YEAR 7

The mean performance of the Year 7 students was in the transition area from level 3 to level 4. About 92 per cent of the students demonstrated Working Mathematically processes and understandings associated with at least level 3 outcomes and about 61 per cent with at least level 4 outcomes. There was an increase of the number of students working at levels 4 or 5.

There was a decline in overall mean Year 7 performance from 2000 to 2002.

YEAR 10

The mean performance on all strands for the Year 10 group was on the transition area from level 4 to level 5. Nearly 80 per cent of the students demonstrated Working Mathematically processes and understandings associated with at least level 4 outcomes and about 50 per cent with level 5 outcomes or above. The results showed that the percentages of Year 10 students working at levels 5 or 6 increased.

Improvement in Year 10 performance from 2000 to 2002 was observed.

SUB-GROUP PERFORMANCES

GENDER PERFORMANCE

For the first time, over the years of testing, the girls outperformed the boys. They performed better than boys in Years 3, 7 and 10.

There was an improvement in mean performance of both Year 10 (girls and boys) and Year 7 (girls only) groups, from 2000 to 2002.

ABORIGINAL AND TORRES STRAIT ISLANDER STUDENTS (ATSI)

In 2002, as in the previous testing, the performance of the ATSI students was significantly below that of non-ATSI students in Years 3, 7 and 10.

There was an improvement in mean performance of both Year 7 and Year 10 ATSI sub-groups, from 2000 to 2002.

STUDENTS WITH LANGUAGE BACKGROUND OTHER THAN ENGLISH (LBOTE)

Over the years of testing, the mean LBOTE students' performance in Mathematics at Year 3, 7 and 10 remained slightly lower than the mean of the students with English speaking background.

Year 10 LBOTE students' performance improved from 2000 to 2002.

MULTI-LEVEL MODELLING OF THE DATA

Multi-level modelling of the data shows whether gender, ethnicity, the school socio-economic status, the size of the schools, the location of the schools, or other variables, have any significant effect on student achievement. These variables are independent of the teaching and learning program.

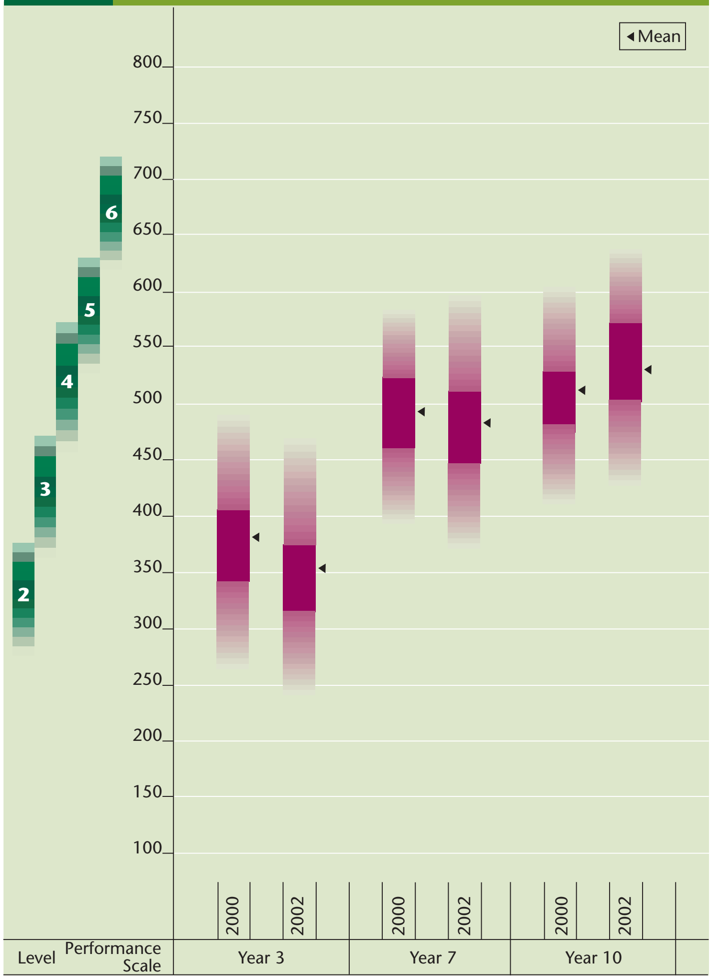
The results of the multi-level analysis showed that:

- students in smaller schools are likely to perform better in year 3;
- students from bigger schools are likely to perform better in Year 10;
- both Year 10 ATSI and non-ATSI sub-groups are likely to achieve relatively better in country schools;
- students are likely to perform better in schools with higher socio-economic status. That is, even when socio-economic status is accounted for, students in these schools are likely to perform better.

Year level	Year of testing	No. of students	Mean score	Standard deviation	Mean level
Year 3	2000	2 412	381	98	Level 3
	2002	1 426	353	85	Level 2
Year 7	2000	1 667	493	86	Level 4
	2002	1 426	482	92	Level 3/4
Year 10	2000	1 158	513	83	Level 4
	2002	1 069	531	91	Level 4

TABLE 1: OVERVIEW OF THE MEAN TEST RESULTS FOR WORKING MATHEMATICALLY IN 2000 AND 2002.

Figure 1. Overview of the Years 3,7 and 10 Performance in Working Mathematically



THE 2002 MATHEMATICS TESTING PROGRAM

MSE 2002 SAMPLE SIZE

Samples of students were drawn at random from Years 3, 7 and 10 in 765 government schools throughout Western Australia. About 5000 students took part in the MSE 2002 study. Because of their relatively small numbers, Aboriginal and Torres Strait Islander (ATSI) students were over-sampled to ensure that valid conclusions could be drawn about their performance.

DATA COLLECTION DESIGN AND MARKING

An important feature of the MSE testing program is its capacity to compare validly performance over time and between year levels. To achieve this, the design of the random-sample-collection process consisted of a combination of common-scale and common-item equating, allowing for the possibility of constructing either two separate scales (one for Content and one for Working Mathematically), or a single scale to be equated to the historical scale. In addition, about 250 students in the sample were asked to do an *equating* test with items already calibrated onto the historical scale (common-case equating).

The administration and supervision of the testing program in each school was the responsibility of the principal. Completed tests were returned to the central office of the Department of Education and Training. Panels of experienced and specially trained teachers marked the test papers. On completion of marking, the results were forwarded to the Australian Council for Educational Research (ACER) in Melbourne for statistical analysis.

DATA ANALYSIS

Data were analysed by *Item Response Theory,* using the Rasch Measurement Model. This model allows a large sample of the population and the range of items on the assessment to be measured on one scale. The students assessed are scaled against each other and against the items on the assessment. The scale provides two key pieces of information:

1. A continuum of skills and understandings, based on the assessment items and arranged in order of increasing difficulty; and
2. The distribution of students in relation to the items.

This scaling allows inferences to be made about the probability of the success of any student on all individual items. Each item can be mapped to the levels of achievement described in the *Outcomes and Standards Framework,* making it possible to identify areas of the scale that relate to each level. In turn, this makes it possible to draw inferences about the level of achievement of groups of students. All descriptions of levels of performance throughout this report can be used to infer levels of achievement. It is therefore possible to infer that a group (for example, a year group) has, at a given point in time, demonstrated a level of performance best characterised by a given outcome statement level.

STATISTICAL SIGNIFICANCE

Apart from describing the performance of particular groups, this report also compares their results with those of the other groups of students. To determine whether the difference between the scores of the groups is a random variation or evidence of an underlying trend, a test of the statistical significance of the difference, at the 0.05 level, has been applied. If the difference has been described as statistically significant at this level, there is a less than 5 per cent chance that it has occurred randomly. In addition, a multilevel modelling of the data, taking into account its hierarchical structure, made it possible to answer questions of the statistical significance of differences in sub-group achievement through the effect of explanatory variables at the school level and the student level.

A difference in performance does not need to be large to be statistically significant, especially with a large sample of students. Therefore, a statistically-significant difference may not be **practically** (or **educationally**) significant. For example, a statistically-significant difference may translate to one score point on a test of 40 items and such a difference will not be of educational significance in a classroom context.

Only differences that are of statistical significance are identified and discussed in this report.

READING THE GRAPHS

PERFORMANCE SCALE

The graphs in this report summarise student achievement in Mathematics. The test results have been mapped onto a performance scale based on the levels of the *Outcome and Standards Framework.* The performance scale is an arbitrary scale running from 0 to 800, in WAMSE (Western Australian Monitoring Standards in Education) units. The scale allows a large sample of the population to be scaled against each other and against the tasks on the assessments. The scale allows comparisons to be made of student performance between year groups and from one year to another (see Figure 2).

LEVELS OF ACHIEVEMENT

Alongside the performance scale are the 'level bars'. The numbers in these bars correspond to the levels of the Outcomes and Standards Framework. Each bar depicts the range of difficulty of tasks in the assessments for that level in the OSF. The areas shown by the graduated shading at the ends of the level bars are where level boundaries overlap. Levels are not absolute as they are indicative of developmental stages and some overlapping of skills occurs at the borders of the levels.

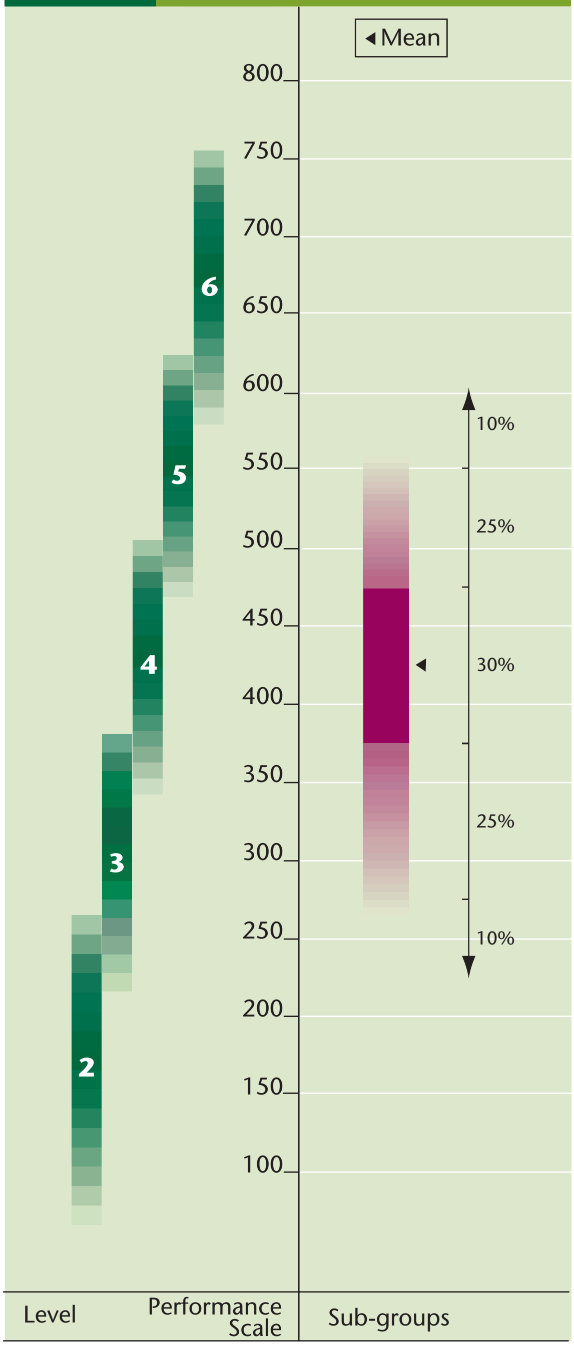
MEAN ACHIEVEMENT

The mean achievement of a sub-group is shown by◀.

SHADED BAR

The shaded column (bar) shows the distribution of 80 per cent of students. The middle, dark-shaded block represents the middle-achieving 30 per cent of students. The lighter-shaded portions, above and below the block, each represent a further 25 per cent of students. Of the remaining 20 per cent of students, 10 per cent achieved above and 10 per cent below the shaded areas. By reading across to the 'levels' on the left-hand side, it is possible to estimate the percentage of students performing at each of the levels. When a group of students is described as demonstrating skills and understandings associated with a particular level, level 4 or above for example, it means that this group of students has demonstrated some of the level 4 outcomes and has achieved most of the outcomes below that level.

Figure 2. Example of a Cigarette Graph



THE MSE 2002 ASSESSMENT INSTRUMENTS

WORKING MATHEMATICALLY

The nature of the MSE 2002 assessment instruments reflected the *Outcomes and Standards Framework* and opinions of teachers and educators on which aspects of Working Mathematically should be assessed in a paper-and-pencil test and what the structure and format of the assessment items should be.

Working Mathematically focuses on the nature of students' ways of thinking and students' problem-solving strategies when working within mathematics. It emphasises the use of a variety of justifications and classroom activities, such as investigating, problem posing and problem solving. These processes could be prompted by a variety of representations: tables, graphs, pictures or verbal descriptions. By their nature, the Working Mathematically outcomes typically demand that students generate their own responses rather than writing an answer.

The item writers and panel members agreed to design a pencil-and-paper test that could be applied widely to large samples, rather than to use techniques that might obtain very rich information (such as interviews or observation) but which would, of necessity, be restricted to smaller samples. It was acknowledged by the panel that not all aspects of Working Mathematically could be addressed through a pencil-and-paper test.

The Working Mathematically items used in the test papers were intended to assess outcomes ranging from level 2 to level 7.

At level 7 students make links between the development and use of mathematical ideas and the conditions and concerns of the individuals and communities which produce them. **They pose, clarify and refine mathematical questions to help guide investigations and use strategies including generalising from one problem situation to another.** *Students provide explanations of why own and others' generalisations "must be true"; and compare the choices between alternative methods and models,* ***including considering assumptions needed and results obtained.***

At level 6 students use familiar mathematics to represent, describe and explain some features of their world. **They partition a problem into sub-problems to help guide its investigation and use strategies including identifying and working on related problems or sub-problems.** *They make generalisations by abstracting common mathematical features from situations or data. They test by making checks of additional cases and understand that only counter-examples are conclusive.* ***They look back and ask if the mathematics chosen was helpful, the assumptions reasonable and the solution good one.***

At level 5 students describe how some familiar mathematical ideas are, or have been used to represent, describe and explain their world. **They extent tasks by asking some original mathematical questions and use strategies including developing systematic approaches.** *They draw on mathematical knowledge to give reasons for conjectures prior to testing them, and refine and modify conjectures as a result.* ***They check working and reasoning, and that answers fit specifications and make sense.***

At level 4 students compare the ways in which familiar mathematics is done or used in own or other communities. **They ask questions to clarify the essential mathematical features of a problem and use strategies including those based on identifying and organising a key information.** *They use examples to support or refute mathematical conjectures* ***and prompted they check that answers are roughly as expected and that methods and answers make sense.***

At level 3 students identify familiar mathematical features inherent in own and other communities. **They pose mathematical questions prompted by specific stimulus or familiar contexts and use problem solving strategies including key information in models, diagrams and lists.** *They understand mathematical conjectures as more than simply a guess. They make straightforward tests of conjectures and discard those that fail the test.* ***When prompted students use alternative ways to check working and choice of method.***

At level 2 students describe some of the ways self, friends and families use mathematical ideas in daily life. **They pose questions. They respond to "What would happen if…?" type of questions about numbers, shapes, data and measurements.** *They test their ideas with at least a single trial. They expect to get the same answer to counting, computation and measuring tasks each time they are carried out* ***and when prompted they check and correct answers.***

Key:

Contextualise Mathematics

Mathematical Strategies

Reasoning Mathematically

Apply and Verify

CONTEXTUALISE MATHEMATICS: ASSESSMENT ITEMS

Students demonstrate that they can Contextualise Mathematics when they provide evidence that they understand that mathematics has its origin in many cultures and is significant in explaining and influencing aspects of their lives.

The assessment items required students to identify some aspects of mathematics inherent in a picture, story or a drawing.

The test item shown in Example 1 required students to identify some familiar mathematical ideas inherent in their world. About 33 per cent of the Year 3 students identified in the picture two different activities involving mathematics.

Example 1:

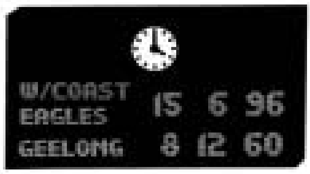
SCOREBOARD

John wrote two different things that involved mathematics in the picture.

1. The height of the bar.
2. The angle of the bar with each post.

Question 15

Below this picture write two **different** things that involve mathematics.



1. You can add up the score for mathematics.
2. You can count the clock timers time.

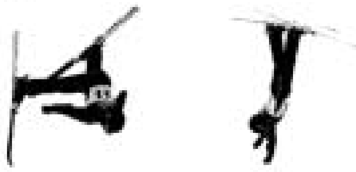
A similar question as the one in the first example was presented to Year 10 students. About 73 per cent of them identified 3 ways in which mathematics is involved in the aerial skiing.

Example 2:

AERIAL SKIING

At the 2002 Winter Olympics, Alisa Camplin won a gold medal in the Women's Aerials Freestyle Skiing.

This event requires the contestants to ski down a slope onto a short ramp. This launches them into the air where they do acrobatics before landing back on their skis lower down the slope.



Question 8

List **three** ways in which mathematics is involved in this event.

- The angle of the slope
- The hight of the ramp
- Working out thier scores.

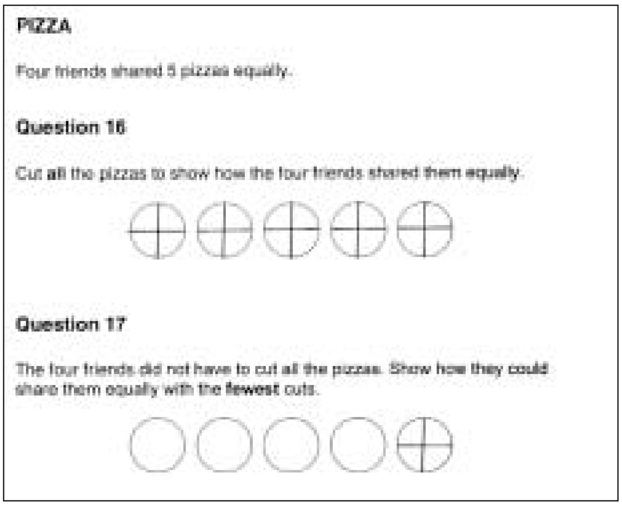
MATHEMATICAL STRATEGIES: SUB-STRAND DESCRIPTION

Students demonstrate that they can use Mathematical Strategies when they provide evidence that they identify and use general heuristic strategies that help them to solve mathematical problems.

The emphasis of this sub-strand is on problem-posing activities as a part of students' problem-solving strategies, particularly in the early levels. This implies that the questions need to be in a free response format, requiring students to generate and explain their reasoning.

The test items shown in Example 3 required students to devise, apply and improve a problem-solving strategy in a number context. The first question was completed correctly by 70 per cent of the Year 3 students. Nearly 40 per cent of the students provided a correct response to the second question.

Example 3:



PIZZA

Four friends shared 5 pizzas equally.

Question 16

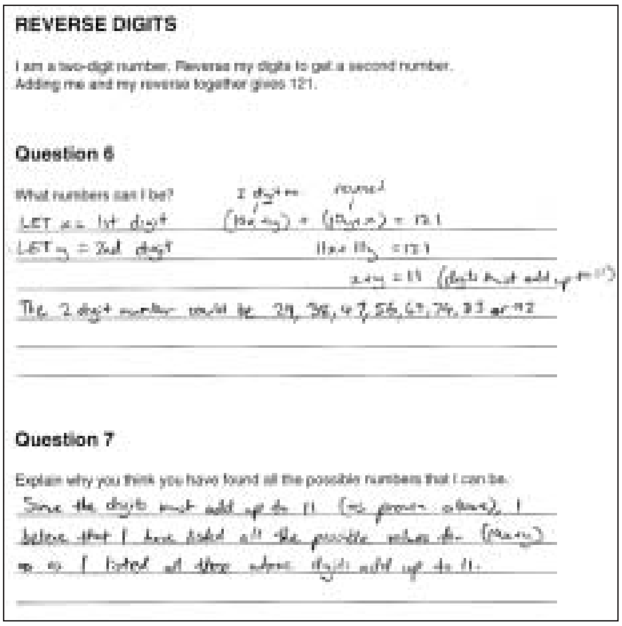
Cut all the pizzas to show how the four friends shared them equally.

Question 17

The four friends did not have to cut all the pizzas. Show how they could share them equally with the fewest cuts.

The questions presented in the next example required the use of problem-solving strategies, including those based on developing systematic approaches, and justifying the results. Nearly 30 per cent of the students listed some, but not all possible combinations. About 25 per cent of the Year 10 students listed all the possible combinations (29 and 92; 38 and 83; 47 and 71; 56 and 65) and about 12 per cent of the students provided a correct justification.

Example 4:.



REVERSE DIGITS

I am a two-digit number. Reverse my digits to get a second number. Adding me and my reverse together gives 121.

Question 6

What numbers can I be?

LET x = 1st digit

LET y = 2nd digit

The 2 digit number could be 29, 38, 47, 56, 65, 74, 83 or 92

Question 7

Explain why you think you have found all the possible numbers that I can be.

Since the digits must add up to 11 (as proven above), I believe that I have listed all the possible values ... so I listed all those whose digits add up to 11.

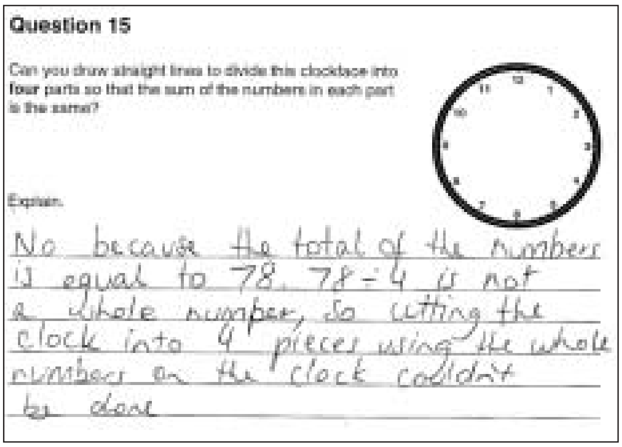
REASON MATHEMATICALLY: SUB-STRAND DESCRIPTION

Students demonstrate that they can Reason Mathematically when they make and test conjectures and justify conclusions reached.

Identification of unprompted reasoning as a core of the Reason Mathematically ability required problem items with open-response formats for this sub-strand. Research has shown that higher outcomes are associated with a tendency to explain well without being asked to do so. Asking students to comment on the reasoning of fictional students (described in an appropriate scenario) was used to prompt students with less developed reason-mathematically skills.

In the next example Years 7 and 10 students were asked to divide the clockface into four parts so, that the sum of the numbers in each part is the same. The example required students to explain their reasoning why something could or could not be done. About 10 and 13 per cent respectively of the Years 7 and 10 students provided partial or complete justifications as to why such division is not possible.

Example 5:



Question 15

Can you draw straight lines to divide this clockface into four parts so that the sum of the numbers in each part is the same?

Explain.

No because the total of the numbers is equal to 78. 78 ÷ 4 is not a whole number, so cutting the clock into 4 pieces using the whole numbers on the clock couldn't be done

APPLY AND VERIFY: SUB-STRAND DESCRIPTION

Students demonstrate that they can Apply and Verify when they choose techniques and models that fit the constraints of a situation and they check and verify their use. Research studies have shown that the issue of prompting is a key: lower level outcomes involve behaviours that are evident on prompting, where higher-level outcomes (Level 5 or higher) involve unprompted behaviour. Some Apply and Verify test items required students, for example, to indicate which answers were not reasonable and to justify why.

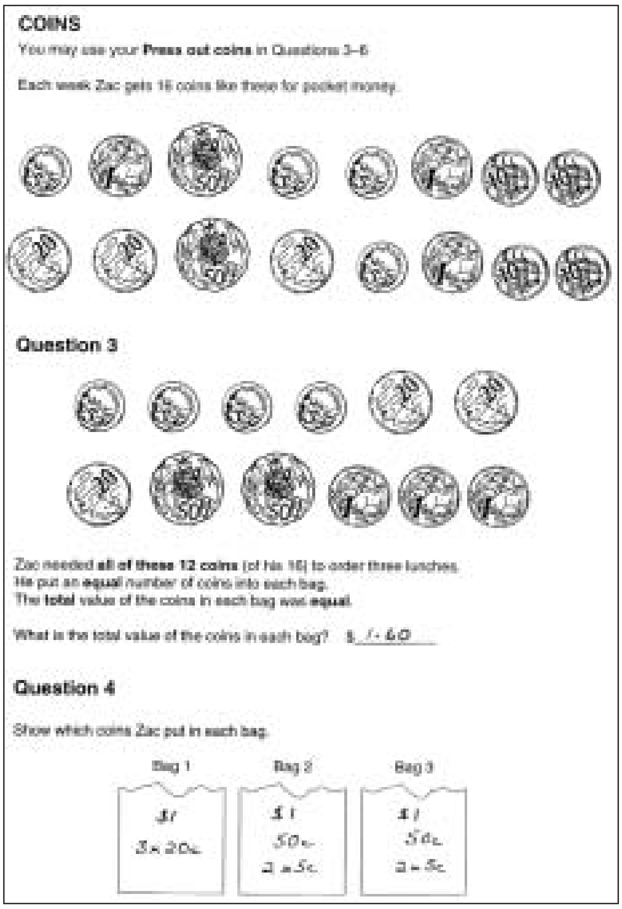
They were also asked to check whether "answers" fitted the specifications of a problem situation and to explain the reasons when they didn't.

The first question in Example 6 required students to find a total amount of money presented in coins and to divide that amount evenly into four parts. About 66 per cent of the Year 7 students provided a correct answer.

The second question involved a distribution of the 12 coins into four amounts of $1.60 each, by using 4 coins for each bag. Nearly 55 per cent of the Year 7 students showed the correct distribution of the coins.

The third part required students to provide convincing arguments that the answer fits the given specification. About 18 per cent of the Year 7 students justified the distribution of the coins.

Example 6:



COINS

You may use your **Press out coins** in Questions 3–6

Each week Zac gets 16 coins like these for pocket money.

Question 3

Zac needed **all of these 12 coins** (of his 16) to order three lunches.
He put an **equal** number of coins into each bag.
The **total** value of the coins in each bag was **equal**.

What is the total value of the coins in each bag? $ 1.60

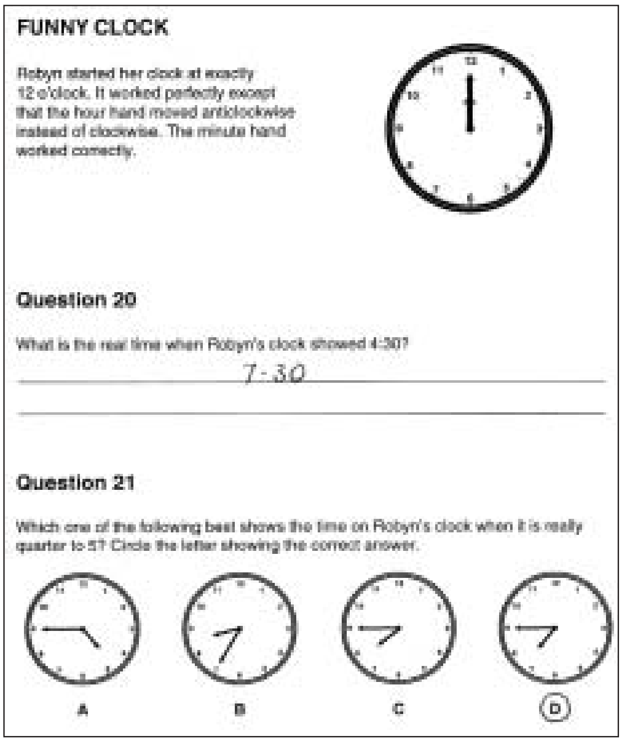
Question 4

Show which coins Zac put in each bag.

Bag 1	Bag 2	Bag 3
$1 3×20c	$1 50c 2×5c	$1 50c 2×5c

The next two questions asked Year 10 students to show and read the time on a clock with the hour hand moving in an anticlockwise direction. Question 20 was completed by 10 per cent of the Year 10 students, and Question 21 by 42 per cent. The most common error for Question 21 was C (27 per cent of the students did not realise that the hour hand should rotate in an anticlockwise direction).

Example 7:



FUNNY CLOCK

Robyn started her clock at exactly 12 o'clock. It worked perfectly except that the hour hand moved anticlockwise instead of clockwise. The minute hand worked correctly.

Question 20

What is the real time when Robyn's clock showed 4:30?

7:30

Question 21

Which one of the following best shows the time on Robyn's clock when it is really quarter to 5? Circle the letter showing the correct answer.

A B C (D)

Student Performance

WORKING MATHEMATICALLY

YEAR 3

Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1426	353	85	Level 2
Girls	699	358	86	Level 2
Boys	713	348	84	Level 2
ATSI	299	294	87	Level 2
LBOTE	180	340	93	Level 2

TABLE 2: YEAR 3 MEAN SCORES ON THE WORKING MATHEMATICALLY STRAND BY SUB-GROUPS.

The mean performance of the Year 3 students was within level 2 Working Mathematically outcomes.

About 80 per cent of the Year 3 students displayed Working Mathematically processes and understandings associated with at least level 2 outcomes and about 40 per cent with at least level 3 outcomes. Approximately 11 per cent demonstrated processes and understandings associated with outcomes at level 4 or above.

About 58 percent of the ATSI students demonstrated Working Mathematically processes and understandings associated with at least level 2 outcomes and about 17 percent with at least level 3 outcomes.

Figure 3. Year 3 Sub-group Performance in Working Mathematically

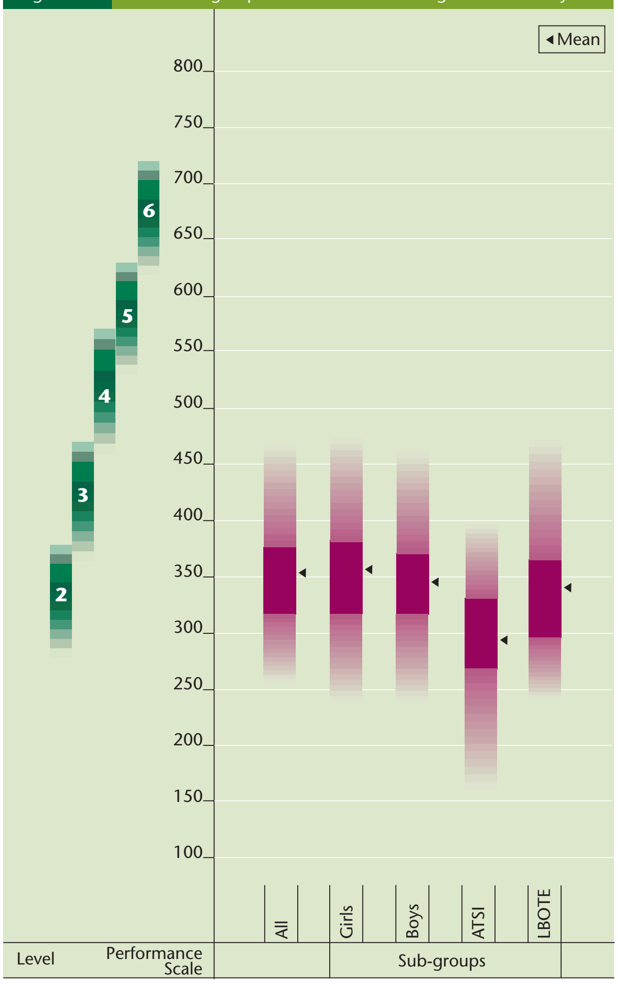
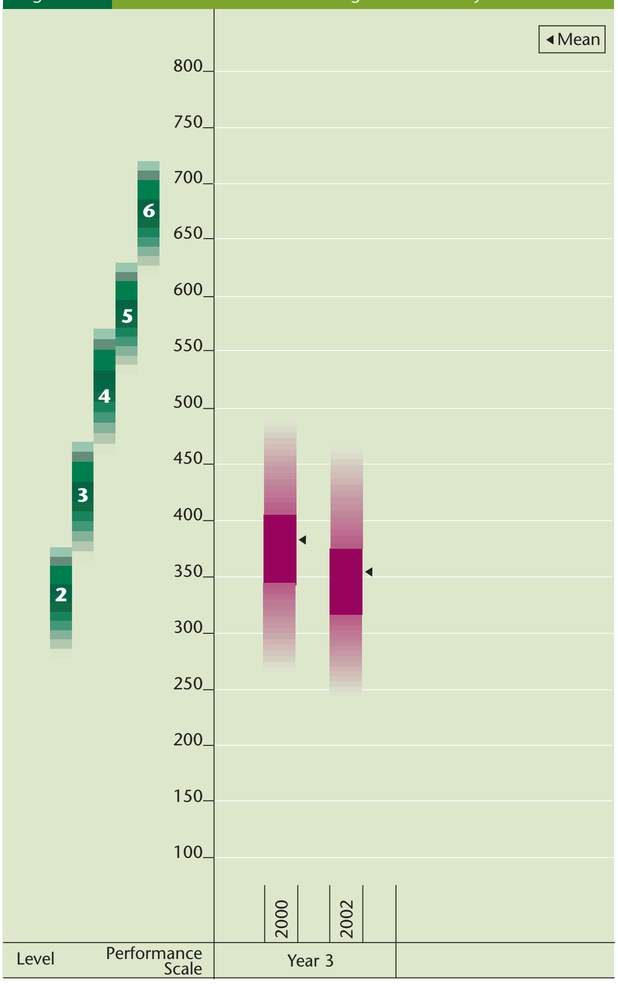


Figure 4. Year 3 Performance in Working Mathematically



YEAR 7

Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1426	482	92	Level 4
Girls	699	488	92	Level 4
Boys	713	476	91	Level 4
ATSI	275	402	99	Level 3
LBOTE	112	466	104	Level 4

TABLE 3: YEAR 7 MEAN SCORES ON THE WORKING MATHEMATICALLY STRAND BY SUB-GROUPS.

The mean performance of the Year 7 students was within level 4 Working Mathematically outcomes. About 92 per cent of the students demonstrated processes and understandings associated with at least level 3 outcomes and about 60% with at least level 4 outcomes. Approximately 28 per cent demonstrated Working Mathematically processes and understandings associated with level 5 outcomes or above.

The mean performance of the ATSI group was at level 3. About 73 percent of the ATSI students demonstrated Working Mathematically processes and understandings associated with at least level 3 outcomes and about 24 percent with at least level 4 outcomes.

Figure 5. Year 7 Sub-group Performance in Working Mathematically

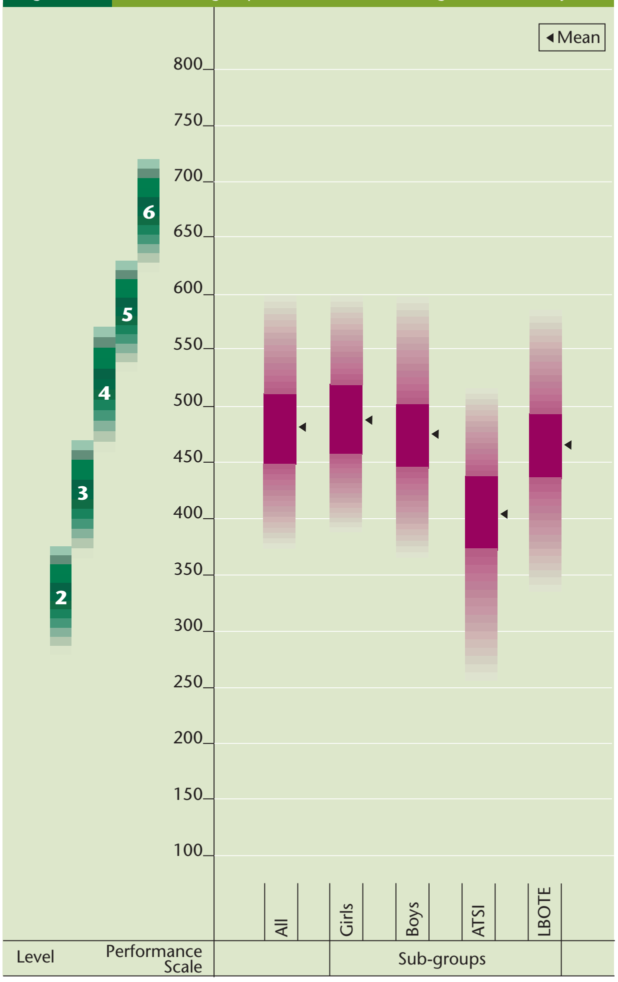
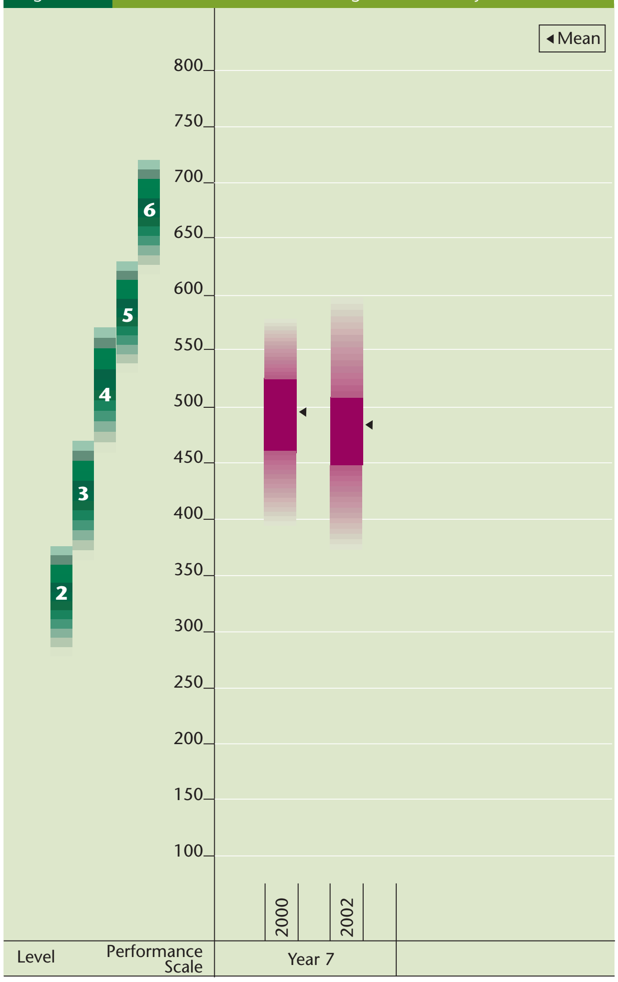


Figure 6. Year 7 Performance in Working Mathematically



YEAR 10

Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1069	531	91	Level 4/5
Girls	516	535	88	Level 4/5
Boys	550	527	93	Level 4/5
ATSI	210	445	95	Level 3
LBOTE	134	512	106	Level 4

TABLE 4: YEAR 10 MEAN SCORES ON THE WORKING MATHEMATICALLY STRAND BY SUB-GROUPS.

The mean Working Mathematically performance for the Year 10 group was in the transition area from level 4 to level 5. Nearly 80 per cent of the students demonstrated Working Mathematically processes and understandings associated with at least level 4 outcomes.

About 41 percent of the ATSI students demonstrated Working Mathematically processes and understandings associated with at least level 4 outcomes and about 18 percent with at least level 5 outcomes.

There was an improvement in Year 10 performance from 2000 to 2002.

Figure 7. Year 10 Sub-group Performance in Working Mathematically

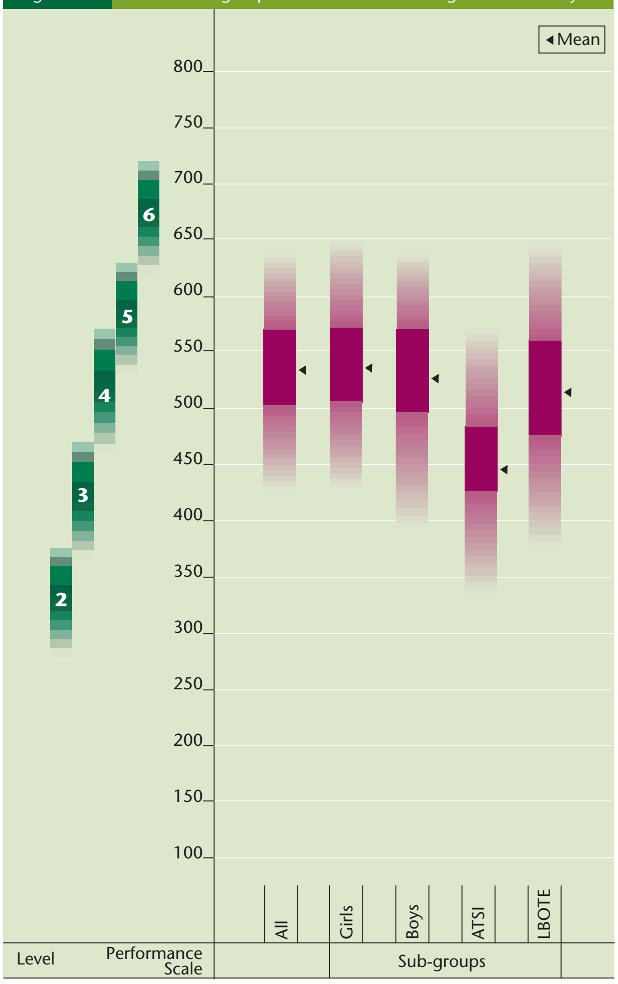
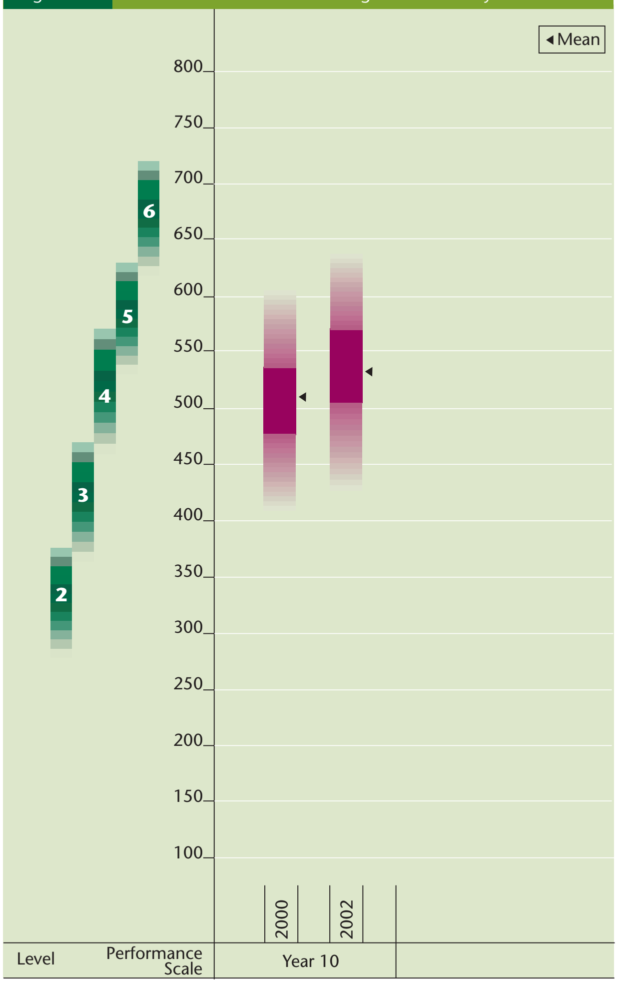


Figure 8. Year 10 Performance in Working Mathematically



SPACE

The strand requires students to describe and analyse mathematically the special features of objects, environments and movements.

The items used in the test paper intended to assess outcomes ranging from level 4 to level 6.

At level 6 students visualise, sketch and describe paths and regions which satisfy provided conditions. They use networks and other diagrams to represent the order of, and paths between locations. **They interpret and meet specifications requiring the accurate construction and placement of figures and objects.** *They visualise, produce and accurately describe specific translations, reflections, rotations and enlargements.* ***They analyse, describe and apply properties of, and relationships between, the classes of figures which can be reasoned about in terms of the properties of triangles and parallel and intersecting lines.***

At level 5 students use coordinates, bearings and scale on maps and plans. They identify the essential features of a location or arrangement needed to serve a purpose. **They visualise and make models of 3D shapes and arrangements and produces conventional mathematical drawings of them.** *They visualise and sketch the effect of straightforward translations, reflections, rotations and enlargements of figures and objects.* ***They analyse, describe and apply distinguishing features of common classes of mathematical figures and objects.***

Level 4 students use distance, direction and grids on maps and plans. **They attend to the shape, size and placement of parts when matching, making and drawing things**, *recognising and using rotation, reflections and translations in arrangements and patterns.* ***They select, describe and compare figures and objects on the basis of spatial features.***

Key:
Represent Location
Represent Shape
Represent Transformations
Reason Geometrically

Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1074	524	98	Level 3/4
Girls	528	525	99	Level 3/4
Boys	543	523	98	Level 3/4
ATSI	205	463	102	Level 3
LBOTE	136	516	102	Level 3/4

TABLE 5: YEAR 10 MEAN SCORES ON THE SPACE STRAND BY SUB-GROUPS.

Sub-group	Level 4 or above	Level 5 or above	Level 6 or above
All	53	28	9
Girls	54	27	9
Boys	51	28	8
ATSI	30	7	2
LBOTE	46	25	10

TABLE 6: PERCENTAGES OF YEAR 10 STUDENTS WORKING AT A SPECIFIED LEVEL OR ABOVE.

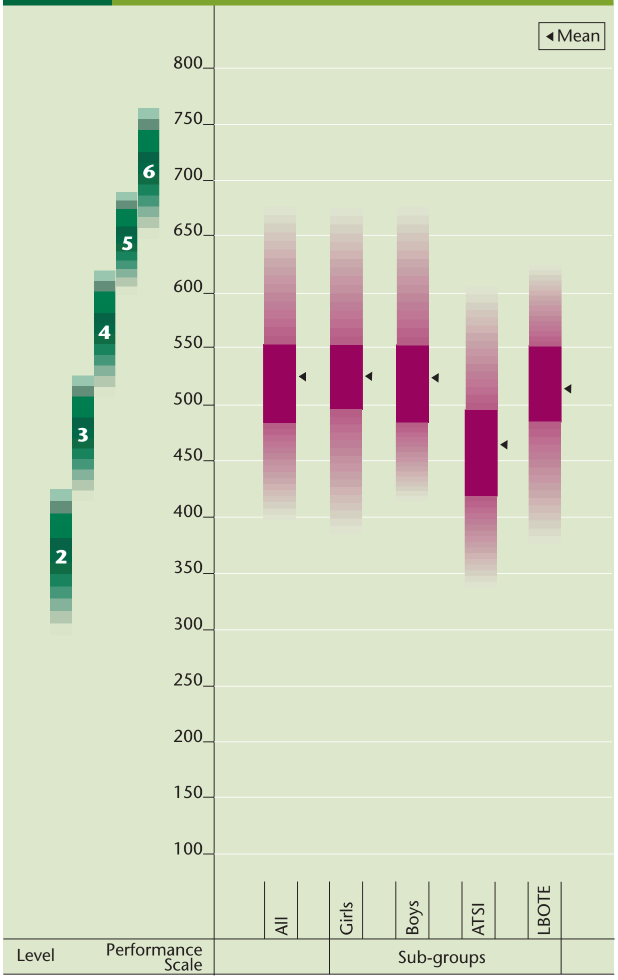
About 53 per cent of the Year 10 students demonstrated skills and understandings associated with at least level 4 Space outcomes, while 28 per cent showed skills and understandings associated with at least level 5, and approximately 9 per cent demonstrated some aspects of at least level 6 outcomes.

The mean performance of the Year 10 group indicated skills and conceptual understandings characteristic of the transition between level 3 and level 4.

The differences between the ATSI and Non-ATSI students were statistically significant.

The mean performance in Space for all sub-groups, except for the LBOTE students, was significantly higher in 2002 than in 2000.

Figure 9. Year 10 Performance in Space by Sub-groups

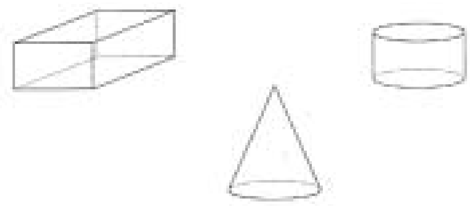


The test item presented in Example 8 drew on students' recognition of conventional drawings of 3D shapes and the use of the basic conventions for drawing them. About 73 per cent of Year 10 students provided the correct response – a diagram of a square-based pyramid.

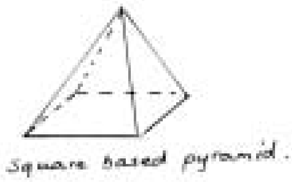
Example 8:

Question 15

Laura has the following solids on her table: a cone, a cylinder, a pyramid with a square base, and a rectangular prism. Here are her drawings of three of these solids.



In the space below, name and draw her fourth solid.



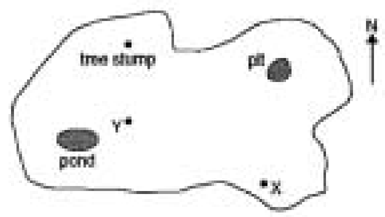
The items presented in Example 9 attempted to assess students' skills in reading informal maps, determining location of specified objects, and using knowledge of scale to find distance between objects on maps.

About 77 per cent of Year 10 students provided a correct response to the first question. Nearly 50 per cent of the students determined the distance between the tree stump and point X correctly.

Example 9:

The following information is needed for Questions 33 and 34.

Jim is at the tree stump on this map. He thinks there is buried treasure at the point marked X. The map is drawn to scale.



Question 33

In which compass direction should Jim walk to reach X?

South east

Question 34

It is 100 m from the tree stump to the point marked Y. Approximately how far (to the nearest 10 m) does Jim need to walk to reach X?

240m.

MEASUREMENT

The measurement outcomes require students to demonstrate the use of direct and indirect measurement and estimation skills to describe, compare, evaluate, plan and construct.

The difficulty of the items used in the test ranged from level 3 to level 6.

At level 6 students decide what measurements are needed, **make or collect measurements to planned levels of accuracy and integrate measurement information from several sources in order to complete a practical task.** *They estimate in situations in which it is sensible to do so and judge whether estimates and measurement are reasonable.* ***They understand and apply length, area and volume relationships for polygons and circles, prisms and pyramids. They understand and use similarity and Pythagoras' theorem to solve problems.***

At level 5 students take purpose and practicality into account when selecting attributes, units and instruments for measuring things. **They use a range of whole number and decimal scales for measuring.** *They make sensible estimates of length, area, mass, capacity and time in standard units.* ***They understand and apply length, area and volume relationships for shapes based on rectangles and rectangular prisms. They understand and use scale factors and the effect of scaling linear dimensions on lengths, areas and volume of figures.***

At level 4 students select appropriate attributes, distinguish perimeter from area and time from elapsed time. **They measure area by counting uniform units and measure length, mass, capacity, time and angle, reading whole-number scales.** *They use the known size of familiar things to help make and improve estimates,* ***understand relationships involving the perimeter of polygons, the area of regions based on squares and the volume of prisms based on cubes, and use these for practical purposes. They understand and use scale factors for straightforward tasks.***

At level 3 students realise that using a uniform unit repeatedly to match an object gives a measure of the size and that a common unity is necessary to compare two things. **They directly and indirectly compare and order things by length, area, capacity, mass, time and angle;** *They make sensible numerical estimates using units that can be seen or handled.* ***They understand and measure perimeter directly and use straightforward arithmetic to determine perimeters and elapsed time. They attend informally to scale when making and using plans, maps and models.***

Key:
Understand Units
Direct Measure
Estimate
Indirect Measure

Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1074	528	99	Level 4
Girls	528	525	97	Level 4
Boys	543	531	100	Level 4
ATSI	205	457	87	Level 3/4
LBOTE	136	527	103	Level 4

TABLE 7: YEAR 10 MEAN SCORES ON THE MEASUREMENT STRAND BY SUB-GROUPS.

About 81 per cent of the Year 10 students demonstrated skills and understandings associated with at least level 4 Measurement outcomes, while 45 per cent showed skills and understandings associated with at least level 5, and approximately 15 per cent demonstrated some aspects of at least level 6 outcomes.

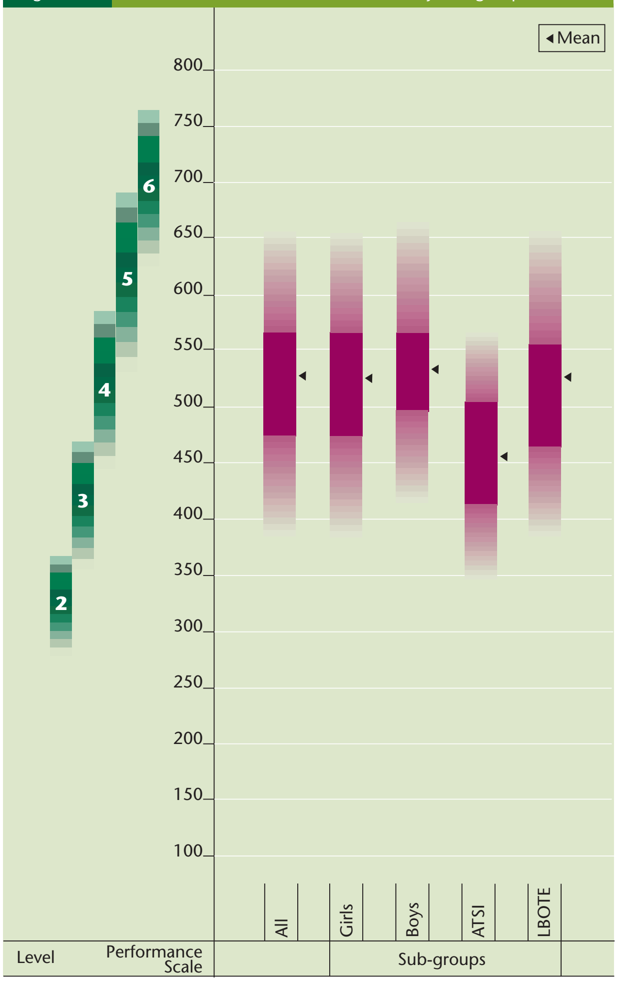
The mean Measurement performance for the Year 10 group was on the transition area from level 4 to level 5.

The improvement in mean performance of all sub-groups was statistically significant from 2000 to 2002.

Sub-group	Level 4 or above	Level 5 or above	Level 6 or above
All	81	45	15
Girls	81	43	14
Boys	81	47	16
ATSI	54	17	2
LBOTE	73	50	14

TABLE 8: PERCENTAGES OF YEAR 10 STUDENTS WORKING AT A SPECIFIED LEVEL OR ABOVE.

Figure 10. Year 10 Performance in Measurement by Sub-groups



The test item presented in Example 10 required students to distinguish perimeter from area and choose units of sensible size for calculating area. About 87 per cent of the Year 10 students selected the correct response.

Example 10:

Question 5

Leila wants to paint a room of her house. To work out how much paint is required, she needs to give the paint shop an estimate of the area to be painted.

Leila would be best advised to give the paint shop an estimate of the area to be painted in

A square centimetres.
B cubic centimetres.
(C) square metres.
D cubic metres.

The question presented in Example 11 attempted to measure students' skills and understandings in applying measurement relationships to the estimation of the length of an object. About 60 per cent of the Year 10 students responded correctly with *C*. Nearly 20 per cent incorrectly selected *B*.

Example 11:

Question 23

Lorraine saw some earrings in a shop window. Each earring was made from coated wire and looked like this:

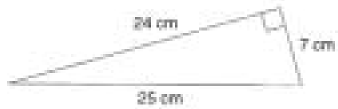
About what length of wire was used to make one earring?

A Less than 5 cm.
B Between 5 cm and 10 cm.
(C) Between 10 cm and 20 cm.
D More than 20 cm.

Calculating the area of a right-angled triangle is a core mathematical skill for the students who have achieved level 5. About 34 per cent of Year 10 students provided the correct answer to this question.

Example 12:

Question 9



What is the area of the triangle? Show your working.

A△ = ½ bh
= ½ × 7 × 24
= 84 cm²

The test item presented in Example 13 attempted to assess students' ability to integrate and use measurement information from several sources in order to complete a practical task. About 29 percent of the Year 10 students provided a correct response.

Example 13:

Question 31

In January, Perth time is 3 hours behind Melbourne time.

Kylie flew from Perth to Melbourne and the trip took 3 hours and 15 minutes.

Kylie's return flight left Melbourne at 6.20 am (Melbourne time) on January 9 and took 50 minutes longer than her flight to Melbourne. Flight = 4h 5m

What was the time in Perth when Kylie arrived back?

Flight to Perth – 4 hours 5 min
Left Melbourne at 6.20am (Melbourne time)
Arrive Perth at 10.25am (Melbourne time)
Arrive Perth at 7.25am (Perth time)

The test item presented in Example 14 required students to use scale factors and understand the effect of scaling the linear dimensions on volumes of objects. About 10 per cent of Year 10 students selected the correct response *D*. The most common error was associated with response A (i.e. *the volume will increase 2 times*).

Example 14:

Question 36

Phillipa has a small box which she can completely fill with exactly 12 centicubes (cubes with side 1 cm). She constructs a second box which is an enlargement of her first box, with all sides enlarged by a scale factor of 2.

If Phillipa completely fills her enlarged box with centicubes, the number she will need is

A 24
B 48
C 72
(D) 96

CHANCE AND DATA

In Chance and Data students demonstrate their knowledge of chance and data handling processes in dealing with data and with situations in which uncertainty is involved.

The difficulty of the test items on this strand ranged from level 4 to level 6.

At level 6 students estimate probabilities based on primary or secondary data collection and assign probabilities for one- and two-stage events. **They plan experiments, surveys and secondary data collection, collaboratively and independently. They check that data are recorded and organised correctly.** *They display and summarise data to show location and variability in order to compare data sets and to show relationships in one data set.* ***They interpret, make comparisons and describes relationships in collected and published data from tables, diagrams, plots, graphs, prose, summary statistics and databases, distinguishing sample and population data.***

At level 5 students interpret and make numerical statements of probability. **They collaborate to plan and refine survey questions and other observation methods for one-variable and two-variable. They collect and record data, including databases.** *They display one-variable and two-variable data in tables and plots and summarise data with fractions, percentages, means and medians.* ***They read and make sensible statements about trends and patterns in the data in tables, diagrams, plots, graphs and summary statistics, and comment on data collection processes and results.***

At level 4 students place events in order from those least to those most likely to happen. **They collaborate with peers to plan what data to collect and how to classify, sequence and tabulate data to answer particular questions.** *They display data using simple scales on axes and some grouping, and summarise data with simple fractions, highest, lowest and middle scores, and means.* ***They read and make sensible statements about the information provided in tables, diagrams, line and bar graphs, fractions and means and comment on how well the data answers questions.***

Key:
Understand Chance
Collect and Organise Data
Summarise and Represent Data
Interpret Data

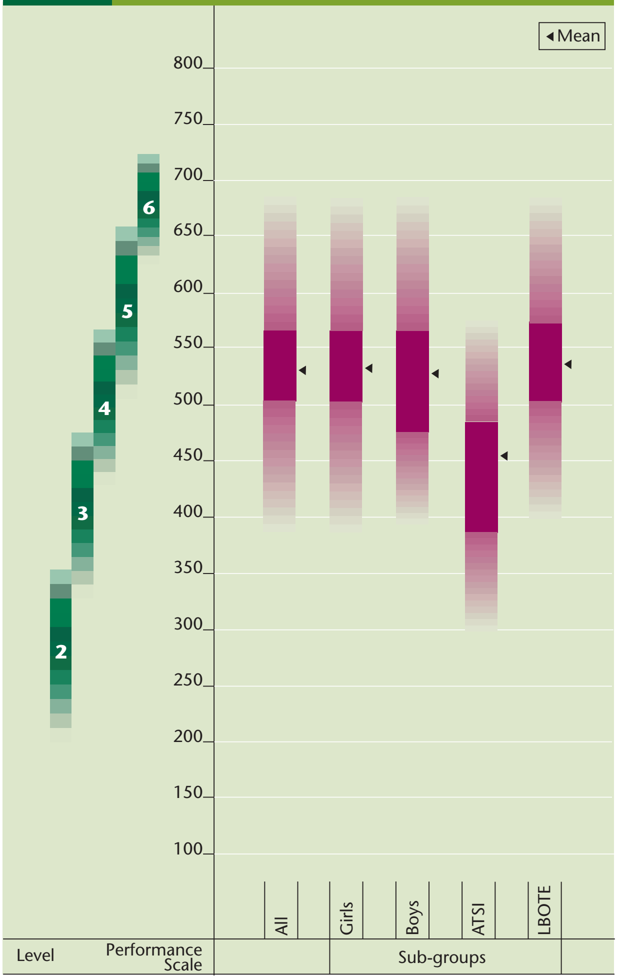
Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1074	530	100	Level 4
Girls	528	531	96	Level 4
Boys	543	528	104	Level 4
ATSI	205	455	99	Level 3/4
LBOTE	136	535	97	Level 4

TABLE 9: YEAR 10 MEAN SCORES ON THE CHANCE & DATA STRAND BY SUB-GROUPS.

Sub-group	Level 4 or above	Level 5 or above	Level 6 or above
All	84	60	13
Girls	85	60	12
Boys	82	60	14
ATSI	62	30	5
LBOTE	88	61	16

TABLE 10: PERCENTAGES OF YEAR 10 STUDENTS WORKING AT A SPECIFIED LEVEL OR ABOVE.

Figure 11. Year 10 Performance in Chance and Data by Sub-groups



About 84 per cent of the Year 10 students demonstrated skills and understandings associated with at least level 4 Chance & Data outcomes, while 60 per cent showed skills and understandings associated with at least level 5, and approximately 13 per cent demonstrated some aspects of at least level 6 outcomes.

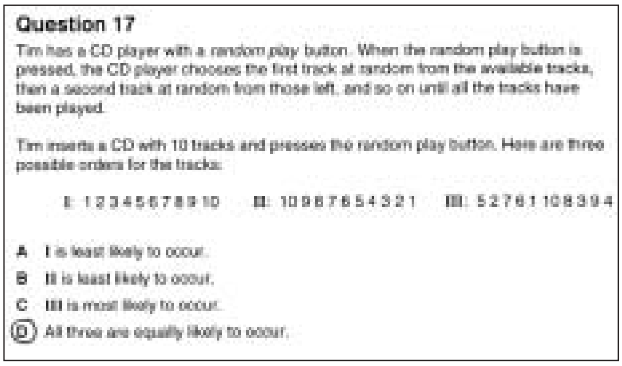
The mean performance of this group indicated skills and understandings characteristic of level 4 outcomes.

There was significant improvement in all Year 10 sub-group performances from 2000 to 2002.

The question presented in Example 15 required students to place events in order from those least likely to those most likely to happen on the basis of numerical information about the events.

Nearly 44 per cent of the Year 10 students selected the correct response.

Example 15:



Question 17

Tim has a CD player with a random play button. When the random play button is pressed, the CD player chooses the first track at random from the available tracks, then a second track at random from those left, and so on until all the tracks have been played.

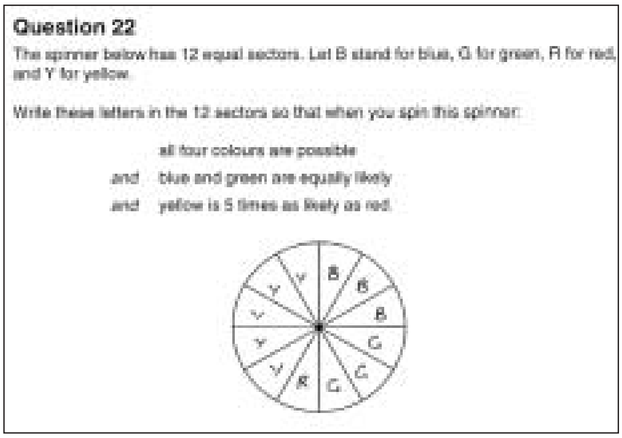
Tim inserts a CD with 10 tracks and presses the random play button. Here are three possible orders for the tracks:

I: 1 2 3 4 5 6 7 8 9 10   II: 10 9 8 7 6 5 4 3 2 1   III: 5 2 7 6 1 10 8 3 9 4

A I is least likely to occur.
B II is least likely to occur.
C III is most likely to occur.
(D) All three are equally likely to occur.

The test item illustrated in the next example asked students to design a spinner that fits a specified probability. About 55 per cent of the Year 10 students provided a correct response.

Example 16:



Question 22

The spinner below has 12 equal sectors. Let B stand for blue, G for green, R for red, and Y for yellow.

Write these letters in the 12 sectors so that when you spin this spinner:

all four colours are possible
and blue and green are equally likely
and yellow is 5 times as likely as red.

NUMBER

The Number strand requires students to use numbers and operations and the relationships between them efficiently and flexibly.

The test included items with difficulty ranging from level 3 to level 6.

At level 6 students read, write, say and understand the meaning, order and relative magnitude of positive and negative rational numbers and numbers expressed with integer powers. **They use ratios and rates to describe the relationship between two quantities.** *They calculate with positive and negative numbers, decimals, fractions and integer powers using mostly mental strategies including for frequently used fractions and percentages of amounts.* ***They classify number patterns which are linear, square or involve a power of a whole number and interpret, construct and clarify rules for describing them.***

At level 5 students read, write, say and understand the meaning, order and relative magnitude of whole and decimal numbers, negative integers, any fractions, straight-forward ratios and percentages. **They understand the meaning, the use, and the connections between the four operations on whole, decimal and fractional numbers and use this understanding to choose appropriate operations.** *They calculate with whole numbers and decimals, drawing mostly on mental strategies for whole numbers, money and readily visualised fractions.* ***They construct and clarify general rules which describe the relationship between each term in a linear sequence and its position.***

At level 4 students read, write, say, count with and compare whole numbers into the millions and decimals. They read, write, say and understand the meaning of fractions. **They use understanding of the meaning, the use, and the connections between the operations on whole and decimal numbers to choose appropriate operations.** *They calculate with whole numbers, money and measures using mostly mental strategies to add and subtract two-digit numbers and for multiplication and divisions related basic facts.* ***They use, recognise and describe and continue whole and fractional number patterns involving one of the four operations.***

At level 3 students read, write, say, count with and compare whole numbers into the thousands. They read, write, say and understand the meaning of unit fractions. **They use understanding of the meaning, the use, and the connections between the operations on whole numbers to choose appropriate operations.** *They add and subtract whole numbers and money, multiplying and dividing one digit whole numbers, using mainly mental strategies for doubling, halving, adding to 100, and additions and subtractions readily derived from basic facts.* ***They use, recognise and describe whole number patterns involving addition, subtraction and multiplication.***

Key:
Understand Number
Operations
Calculate
Reason About Number Patterns

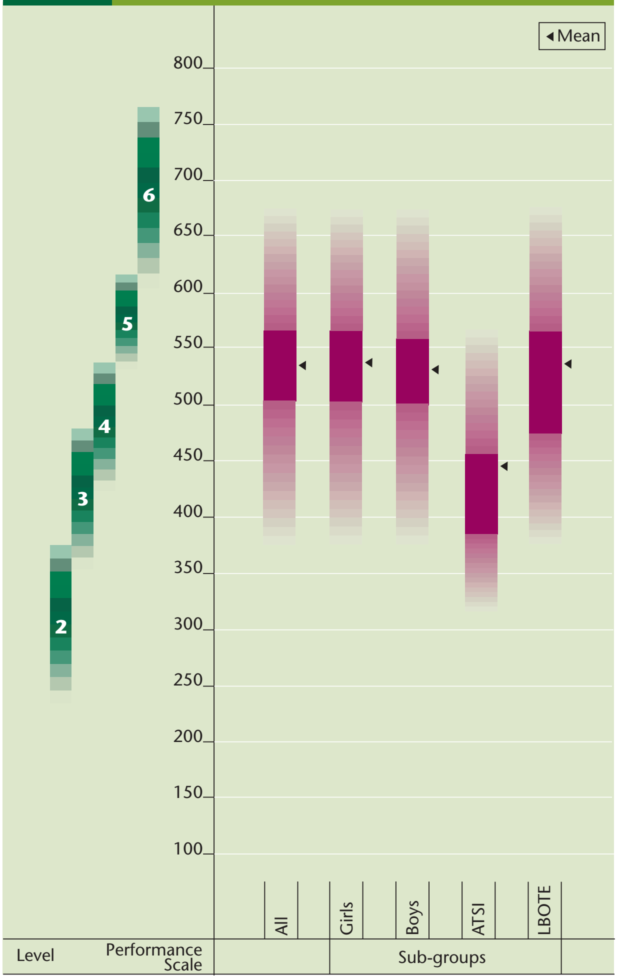
Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1074	533	110	Level 4/5
Girls	528	537	110	Level 4/5
Boys	543	530	110	Level 4/5
ATSI	205	447	98	Level 3/4
LBOTE	136	536	117	Level 4/5

TABLE 11: YEAR 10 MEAN SCORES ON THE NUMBER STRAND BY SUB-GROUPS.

Sub-group	Level 4 or above	Level 5 or above	Level 6 or above
All	85	44	24
Girls	86	48	24
Boys	84	43	21
ATSI	47	17	6
LBOTE	86	44	24

TABLE 12: PERCENTAGES OF YEAR 10 STUDENTS WORKING AT A SPECIFIED LEVEL OR ABOVE.

Figure 12. Year 10 Performance in Number by Sub-groups



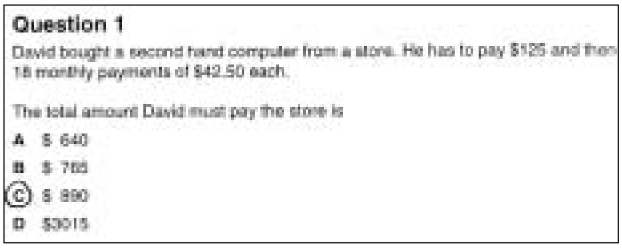
About 85 per cent of the year 10 students showed skills and understandings characteristic of level 4 Number outcomes and about 44 per cent demonstrated skills and processes associated with at least level 5 outcomes.

The mean performance of the Year 10 group on Number indicated skills and conceptual understandings characteristic of levels 4 and 5 outcomes.

There was significant improvement in the mean performance of all sub-groups from 2000 to 2002.

The test item presented in Example 17 required students use whole and decimal numbers to calculate amounts of money in a real life context. About 88 per cent of the students responded correctly with C. The most common misconception was *B* (18 x $42.50).

Example 17:


Question 1

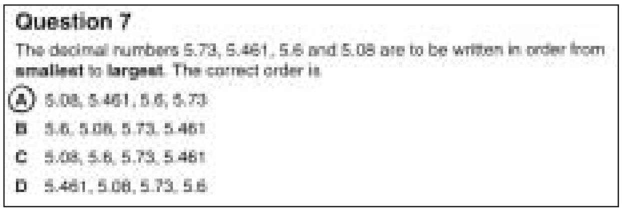
David bought a second hand computer from a store. He has to pay $125 and then 18 monthly payments of $42.50 each.

The total amount David must pay the store is

A $ 640
B $ 765
(C) $ 890
D $3015

The question presented in Example 18 aimed to assess students' understanding of the meaning, order and relative magnitude of decimal numbers. Nearly 67 per cent of the Year 10 students responded correctly with A.

Example 18:


Question 7

The decimal numbers 5.73, 5.461, 5.6 and 5.08 are to be written in order from **smallest** to **largest**. The correct order is

(A) 5.08, 5.461, 5.6, 5.73
B 5.6, 5.08, 5.73, 5.461
C 5.08, 5.6, 5.73, 5.461
D 5.461, 5.08, 5.73, 5.6

The test item presented in the next example attempted to draw on students' mental strategies for calculating with readily visualised fractions.

About 46 per cent of the year 10 students selected A as the correct response. The most common errors were B and D.

Example 19:


Question 15

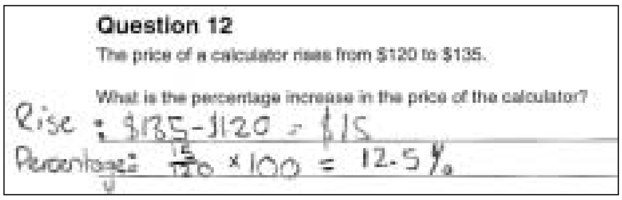
$3\frac{5}{12} - 1\frac{1}{6}$ is equal to

(A) $2\frac{1}{4}$
B $2\frac{1}{3}$
C $2\frac{7}{12}$
D $2\frac{2}{3}$

The question in the next example required students to use percentages in order to express a quantity as part of a whole.

About 62 per cent of the Year 10 students provided a correct response.

Example 20:


Question 12

The price of a calculator rises from $120 to $135.

What is the percentage increase in the price of the calculator?

Rise : $135-$120 = $15
Percentage: $\frac{15}{120}$ ×100 = 12.5%

ALGEBRA

The strand requires student to use algebraic symbols, diagrams and graphs to understand, to describe and to reason.

The test items used in the test intended to assess some level 5 and level 6 outcomes.

At level 6 students use and interpret basic algebraic conventions for representing situations involving a variable quantity. **They plot, sketch and interpret graphs.** *They recognise and represent linear and square relationships in tables, symbols and graphs, and informally describe how one quantity varies with the other.* ***They set up equations to represent one constraint in a situation, solve using 'guess, check and improve' and graphical methods, and solve linear equations using analytical methods.***

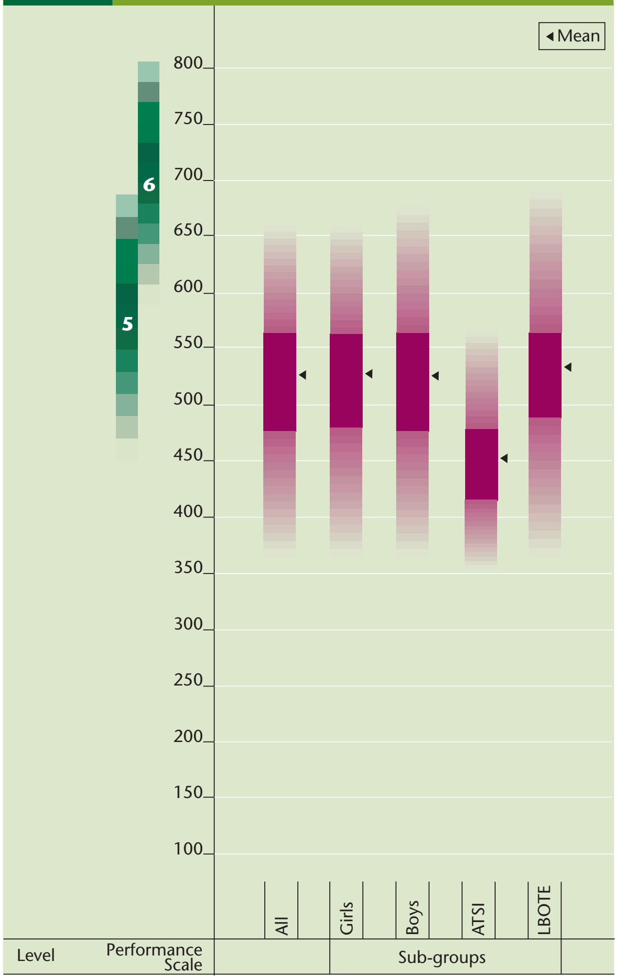
At level 5 students use a letter to represent a variable quantity in an oral or written expression. **They generate and plot data in first quadrant coordinate graphs, describing resulting patterns.** *They interpret graphs which describe the relationship between two quantities in everyday situations.* ***They generate numbers which satisfy a single constraint which is stated in natural language.***

Key:
Understand Symbols
Understand Graphs
Represent Variation
Solve Equations and Inequalities

Sub-group	No of students	Mean score	Standard deviation	Mean level
All	1074	528	104	Level 5
Girls	528	529	100	Level 5
Boys	543	526	109	Level 5
ATSI	205	452	82	Level 5
LBOTE	136	532	114	Level 5

TABLE 13: YEAR 10 MEAN SCORES ON THE ALGEBRA STRAND BY SUB-GROUP.

Figure 13. Year 10 Performance in Algebra by Sub-groups



About 75 per cent of the Year 10 students demonstrated skills and understandings associated with at least level 5 Algebra outcomes, while 28 per cent showed skills and understandings associated with at least level 6 outcomes.

The mean performance of this group indicated skills and understandings characteristic of level 5 outcomes.

There was a significant improvement in Year 10 performance for all sub-groups except for ATSI students from 2000 to 2002.

The test item shown in Example 21 required the use of a letter (as a variable representing a quantity) in an expression involving two operations.

About 60 per cent of Year 10 students responded correctly with *B*. The most common misconception was *C* (corresponds to the order of the numbers in the problem statement).

Example 21:

Question 11

The cooking time for a beef roast is 20 minutes plus 30 minutes per kilogram.

If M is the mass of the roast in kilograms, the total cooking time (in minutes) is

A $50M$

(B) $30M + 20$

C $20M + 30$

D $30(M + 20)$

The question presented in Example 22 required students to simplify a particular algebraic expression.

About 14 per cent of the Year 10 students selected *D* correctly. The most common mistake was associated with an incorrect application of $(a + b)^2$ as $a^2 + b^2$, with 29 per cent of the students selecting *B*.

Example 22:

Question 20

Another form of the quadratic expression $(x + 3)^2 - 2$ is

A $x^2 + 4$

B $x^2 + 7$

C $x^2 + 6x + 4$

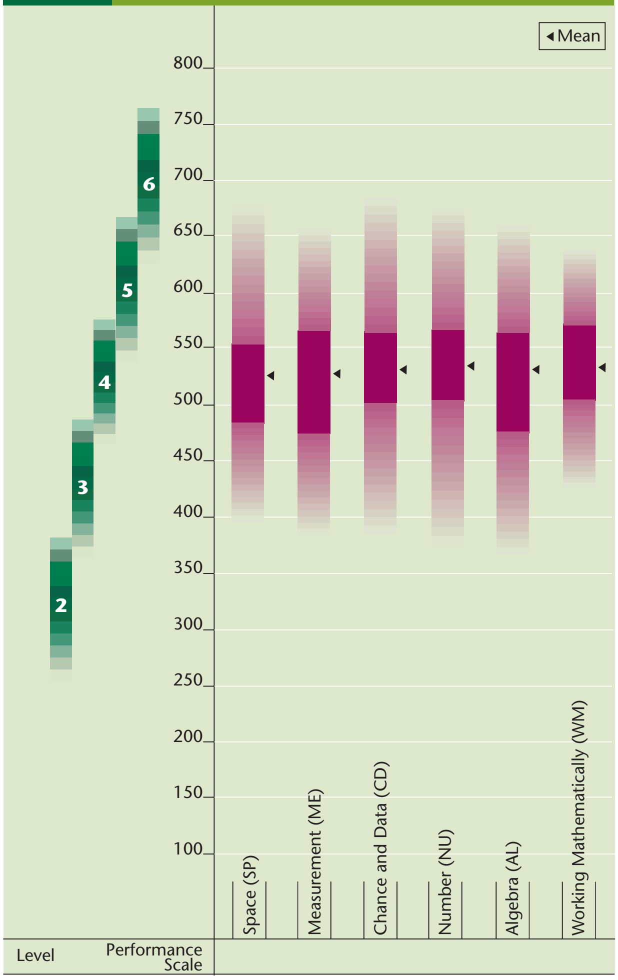
(D) $x^2 + 6x + 7$

YEAR 10 GROUP PERFORMANCE ACROSS STRANDS

	No of students	Mean score	Standard deviation
Space (SP)	1074	524	98
Measurement (ME)	1074	528	99
Chance and Data (CD)	1074	530	100
Number (NU)	1074	533	110
Algebra (AL)	1074	530	100
Working Mathematically (WM)	1069	531	91

TABLE 14: YEAR 10 MEAN PERFORMANCE ACROSS THE STRANDS.

Figure 14. Year 10 Performance Across the Strands by Sub-groups



The distribution of Year 10 results showed a small variation in student mean achievement across the strands.

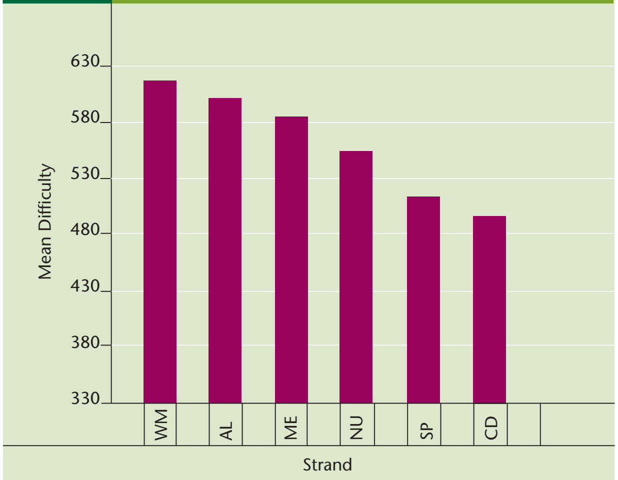
On average, the Year 10 mean achievement, as expressed through the underlying scale, was reasonably even in the Working Mathematically, Measurement, Number, Chance and Data, and Algebra strands, and slightly lower in the Space strand.

Number strand has the widest range of performance.

Strand	Level 4 or above	Level 5 or above
Space (SP)	53	28
Measurement (ME)	81	45
Chance and Data (CD)	84	60
Number (NU)	85	44
Algebra (AL)	NA	75
Working Mathematically (WM)	78	52

TABLE 15. PERCENTAGES OF YEAR 10 STUDENTS WORKING AT LEVELS 4, 5 OR ABOVE.

Figure 15. The mean strand difficulty



Overall, about 76 per cent of the Year 10 students showed skills, understandings and processes characteristic of level 4 or above outcomes. About 45 per cent demonstrated some outcomes at level 5 or above, and about 12 per cent demonstrated at least some level 6 outcomes.

RELATIVE DIFFICULTIES OF THE MATHEMATICS STRANDS

In addition to assessing students' overall mean performance and achievement across the strands, the relative difficulty of the strands was determined. The difficulty of a strand relative to the other strands was defined as the mean difficulty of the items belonging to that strand.

Figure 15 shows the difficulties of the six strands on the WAMSE scale.

The Working Mathematically items administered to the students were found to be, on average, the most difficult. From the content strands, the Algebra items were found to be more difficult than the Chance and Data items.

Caution should be exercised when drawing conclusions from this information. Working Mathematically and Algebra may actually be more difficult strands, or it may be that easier items were selected for Chance and Data, or it may reflect limited curriculum exposure to the Working Mathematically and Algebra strands.