

## Graphing on the TI-83

### Keys used for creating graphs

The keys which are located just under the screen are the main ones used to create and investigate graphs.

**Y=**

This is the key used to enter the expression, or function, which you want to graph.

**WINDOW**

This key allows you to specify the scale for the  $x$ - and  $y$ -axes. This is where you set the maximum values for  $x$  and for  $y$  that you wish to see on the screen.

**ZOOM**

You can zoom in on a graph to show more detail - just like using a magnifying glass. Or you can zoom out to see more of the graph if you need to see the 'big picture'. Other options will be described as they are needed later.

**TRACE**

When you need to investigate how  $x$  and  $y$  values are changing, we use the *trace* facility. It allows you to follow the path of a graph with a cursor, and to read the coordinates of the point where the cursor is located

**GRAPH**

Once you have told the calculator what you want to graph, pressing this key then actually draws the graph.

There are other options printed in yellow above each of these keys. These are accessed by first pressing **2nd** - their use will be explained later.

## 1. Defining functions to be graphed

In order to graph a function, it must first be entered on the function list.

Press  $\boxed{Y=}$  to go to the function list. If there are any functions already defined, you can clear these by moving the flashing cursor (using the  $\boxed{\rightarrow}$   $\boxed{\downarrow}$  etc. keys) to any position of the expression itself (but **not** on the = sign), and press  $\boxed{\text{CLEAR}}$

You can define up to 10 different functions (labelled as Y1, Y2, Y3, etc.) and all or some of these can be graphed simultaneously.

If you wish to leave a function defined but do not want it graphed, then move the flashing cursor to be over the = of that expression, and press  $\boxed{\text{ENTER}}$ . The = sign will no longer be highlighted. To reselect a function to be graphed, move the cursor to the = sign and press  $\boxed{\text{ENTER}}$

## 2. Defining the Window

Unless you specify otherwise, a graph will be displayed on the standard screen. For this the  $x$ -axis is from  $-10$  to  $+10$  in steps of 1 unit, and the same for the  $y$ -axis. You need to be aware of the settings for the axes so that you can interpret the graph, as no scale is written on the axes. You can design the  $x$  and  $y$  axes for a graph by specifying maximum and minimum values that you wish.

Press  $\boxed{\text{WINDOW}}$ . You will see the current information about the axes. You can change any of these settings by overwriting them.

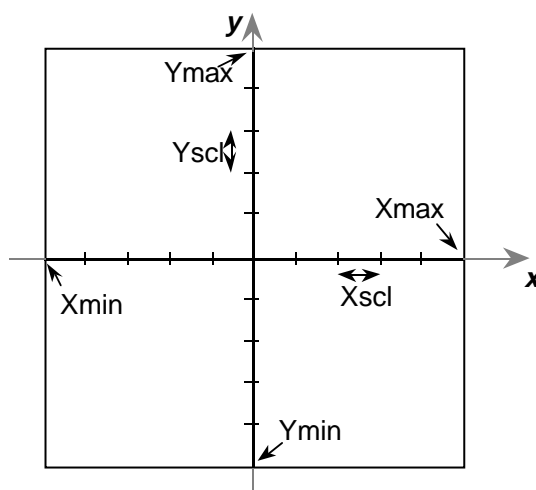
The setting **Xscl** means the **scale** for the marks along the  $x$ -axis; that is, the size of the 'jumps' between the maximum and minimum values. Similarly for **Yscl**.

**Xres** sets the pixel resolution.

If  $Xres = 1$ , the function is graphed at every pixel on the  $x$ -axis.

If  $Xres = 8$ , the function is graphed at every 8th pixel on the  $x$ -axis.

Generally we leave  $Xres = 1$ .



### 3. Graphing the function $y = x^2$

Press  $\boxed{Y=}$

At Y1, press  $\boxed{X T \theta n}$  *This enters the variable  $x$ .*

press  $\boxed{x^2}$  *Now we have the variable squared.*

For the graph of  $y = x^2$ , let's choose the  $x$ -axis to be from  $-4$  to  $+4$ , going up in steps of 1; and for the  $y$ -axis, set Ymin to be  $-1$  (since we know that the curve will be above the  $x$ -axis), and Ymax to be 20, again going up in steps of 1. Press  $\boxed{WINDOW}$  and enter these settings.

Xmin =  $-4$  *Make sure you enter this using the keys  $\boxed{(-)}$  4*

Xmax = 4

Xscl = 1

Ymin =  $-1$  *Enter using  $\boxed{(-)}$  1*

Ymax = 20

Yscl = 1

press  $\boxed{GRAPH}$

The graph of  $y = x^2$  will now be displayed on the setting you have chosen for the axes.

Note: There is an alternative way to define a squared term. After pressing  $\boxed{X T \theta n}$  you could enter  $\boxed{\wedge}$  2. The  $\boxed{\wedge}$  key is used when entering any other powers of  $x$ .

### 4. Tracing the function

A graph can be traced by pressing the  $\boxed{TRACE}$  key.

A flashing cursor ('star') appears on the graph, the coordinates of that point are given at the bottom of the screen. The equation on which the cursor is currently located is shown at the top left of the screen.

Use the  $\boxed{\rightarrow}$  key to move the cursor to the right along the graph. Notice how the coordinates given at the base of the screen of this point change. To move to the left, use the  $\boxed{\leftarrow}$  key. These coordinates are given to the number of decimal places specified using the  $\boxed{MODE}$  key.

To remove the tracing cursor (and its coordinates) press  $\boxed{CLEAR}$

The starting position of the cursor is always at the central value for  $x$  on the screen. Since the window is set for  $-4 \leq x \leq 4$ , the central position is at  $x = 0$ .

## 5. Graphing the function $y = x + 2$

Press  $\boxed{Y=}$

At Y2, press  $\boxed{X T \theta n}$  *This enters the variable  $x$ .*

press  $\boxed{+}$  2

press  $\boxed{\text{GRAPH}}$

The graphs of  $y = x^2$  and  $y = x + 2$  are both displayed.

If we wish to investigate where these two graphs intersect it will be more convenient to view over a smaller range of  $x$  and  $y$  values.

Change these values using

$\boxed{\text{WINDOW}}$  ( $-4 \leq x \leq 4$  and  $-1 \leq y \leq 5$  is a good window to use.)

Press  $\boxed{\text{GRAPH}}$  to return to the graph screen, and then press  $\boxed{\text{TRACE}}$ . The flashing cursor will be on the curve of  $y = x^2$  - as indicated by the equation at the top of the screen. Use the  $\boxed{\uparrow}$  (or  $\boxed{\downarrow}$ ) keys to move the cursor to the graph of Y2 (i.e.  $y = x + 2$ ), and the equation  $Y2 = x + 2$  is now shown at the top of the screen.

To determine the coordinates of the points of intersection, move the cursor (either  $\boxed{\rightarrow}$  or  $\boxed{\leftarrow}$ ), until it appears to be at the point of intersection and then read the coordinates. (One point should be very close to  $x = 2$ ,  $y = 4$ )

## 6. Zooming in and out

In trying to locate the point of intersection of  $y = x^2$  and  $y = x + 2$  when  $x$  is positive (above) you may have noticed that the cursor 'jumps' and you may not be sure of its position. This is when we need to 'zoom in' to magnify the graph.

With the cursor located as close as you can to the point of intersection, which we know will be at (2,4),

press  $\boxed{\text{ZOOM}}$  2

*The number 2 is pressed so that you zoom in.*

*At this stage you will be returned back to the graph, but nothing seems to have changed - yet!*

press  $\boxed{\text{ENTER}}$

*Now you **will** see a close up of the point of intersection!*

Again press  $\boxed{\text{TRACE}}$  and use the cursor keys to locate the point of intersection more accurately, and then zoom in again if you wish.

The keystrokes  $\boxed{\text{ZOOM}}$  3  $\boxed{\text{ENTER}}$  will undo the zooming process. Alternatively,  $\boxed{\text{ZOOM}}$  6  $\boxed{\text{ENTER}}$  will return you to the standard graph window, i.e.  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .

## 7. Evaluating at specific x values

A very convenient feature is the ability to find the y-value for any graph at any particular value for x.

Whilst you have the graphs on the screen, press  $\boxed{2\text{nd}}$   $\boxed{\text{TRACE}}$  and then choose the option *value*.

You will be returned to the graph screen and notice that the cursor is positioned at the bottom of the screen and waiting for you to prompt it with an x value.

Enter -1, say - this means find the y-value of Y1 when  $x = -1$ .

After pressing  $\boxed{\text{ENTER}}$  the value of y is also shown at the bottom of the screen; in this case,  $y = 1$ .

If you press  $\boxed{\uparrow}$  you will notice that the cursor moves to Y2, and you can also evaluate Y2 at  $x = -1$ . Since this will show you that Y2 also is 1 when  $x = -1$ , you have confirmed that the point  $(-1, 1)$  is a point of intersection of the graph.

If you look at the graph, it appears that the other point of intersection is at  $x = 2$ . Try evaluating Y1 and Y2 at  $x = 2$  to confirm that this is the other point of intersection.

You will have probably realised by now that finding these points of intersection has actually given you the solution of the equation

$$x^2 = x + 2 \qquad \text{or} \qquad x^2 - x - 2 = 0$$

## Solving equations

Algebraic equations can be solved graphically by finding where the equation intersects with the  $x$ -axis, or by using an algebraic method. For a quadratic equation, an algebraic method could be by factorizing or using the quadratic formula.

Using a graphics calculator **any** equation can be solved by either graphical or algebraic means.

### Graphical solution of equations

There are two alternatives to use here.

1. You can graph the function and then use the **Trace** facility to zoom in on the coordinates of the  $x$ -intercepts. Enlarge the region around the intercept until you have the desired level of accuracy.
2. You can use the **Calc** facility and ask the calculator to find the roots of any graph.

We will use the function  $f(x) = x^3 - 3x^2 + 2$  to demonstrate both of these methods.

### The 'trace' method

Plot the function  $f(x) = x^3 - 3x^2 + 2$ .

Suggested window settings:

Xmin	=	-2	Ymin	=	-3
Xmax	=	4	Ymax	=	3
Xscl	=	0.5	Yscl	=	0.5

Press GRAPH The graph appears to have one root at  $x = 1$ , and the other two are in the intervals

$$-1 < x < -0.5 \text{ and } 2.5 < x < 3.$$

Press TRACE You will notice that the cursor is at the point

$(1, 0)$ , because  $x = 1$  is the centre of the screen. This confirms that there is one root at  $x = 1$ .

Use the → key to put the cursor as close as possible to the root which is at  $x \approx 2.7$

You will notice that you only need to use the → and ← keys to move the cursor **along** the path of the curve.

Press **ZOOM** 2 **ENTER** **TRACE**

Using **TRACE** ensures that the cursor stays on the curve when either **→** or **←** are pressed. Position it as close as possible to the intersection of the curve and the x-axis.

Continue to press **ZOOM** 2 **ENTER** **TRACE** until you have a value for the x-intercept which is to the required level of accuracy. (To 2 d.p.,  $x = 2.73$ )

To leave this enlarged screen, you can return directly to the standard screen by pressing **ZOOM** 6 **ENTER**. Then reset the window to the settings as given at the beginning.

You can 'unzoom' by progressively pressing **ZOOM** 3 **ENTER**. However this will not take you back to the original window, so the method above is probably the easier one.

## Finding roots of a function

Plot the function  $f(x) = x^3 - 3x^2 + 2$ .

Suggested window settings:

Xmin	= -2	Ymin	= -3
Xmax	= 4	Ymax	= 3
Xscl	= 0.5	Yscl	= 0.5

Press **GRAPH** The graph appears to have one root at  $x = 1$ , and the other two are in the intervals

$$-1 < x < -0.5 \text{ and } 2.5 < x < 3.$$

Press **2nd** **TRACE** This accesses the **Calc** key.

Press 2 This chooses to calculate a **zero** (or root) of the equation.

The graph is displayed with a prompt at the bottom of the screen to enter **Left Bound**. This is a value for  $x$  which is **less** than the actual root.

Use the cursor **→** or **←** to move along the curve to locate an  $x$  value **less** than the required zero. Then press **ENTER**. You will notice that a little black triangle

appears at the top of the screen - this is to let you know where you have selected the lower estimate for the root.

You will now be prompted for an **Right Bound**, so use the cursor to move along the curve to locate an  $x$  value **greater** than the required zero. Press  , another little triangle will appear to show you the interval that you have selected and from which the zero will be determined.

The calculator now asks you for a **Guess**. In other words, guess a value where you think the zero is, but it must be **between** the Left Bound and the Right Bound. Select this guess by moving the cursor back to the left and pressing

The coordinates of this root will now be displayed. The  $y$  coordinate will be 0 or very close to it. The  $x$  coordinate is the value of the zero to an accuracy of as many decimal places set using the  key.

For this example, you should find  $x = 2.7320508$  (to 7 d.p.) is the zero of the equation in the interval  $2.5 < x < 3$ .

Repeat the procedure to find the zero which is in the interval  $-0.5 < x < -1$ .

You should find that this zero is at  $x = -0.7320508$  (to 7 d.p.)

## Algebraic solution of equations

You do not need to actually graph a function to solve it, instead you can choose the **Equation Solver** function from the MATH menu.

Return to the Home Screen by pressing  $\boxed{2\text{nd}} \boxed{\text{MODE}}$  .

Press  $\boxed{\text{MATH}}$  0

*You will see that **Equation Solver** is entered on the screen and the second line is prompting you to enter the expression which is to be solved.*

If there is an equation already defined, press  $\boxed{\uparrow}$  to move up to the Equation solver starting screen.

Enter the expression  $x^3 - 3x^2 + 2$  in the normal way.

Then press  $\boxed{\text{ENTER}}$

*The cursor is now flashing to prompt you for a value for  $x$  which is close to a solution (root) of the equation.*

Key in  $\boxed{3}$  at this stage

*... this tells the calculator that you want a solution which is close to 3.*

Press  $\boxed{\text{ALPHA}} \boxed{\text{ENTER}}$

*This is the command to **Solve** i.e. to ask for the solution close to 3.*

The solution  $x = 2.7320508075\dots$  will now be displayed.

(Since  $\dots$  appears after all of the digits, you can use the cursor keys to find *more* decimal places for the answer. If you press  $\boxed{\rightarrow}$  enough times you will find the answer to 13 decimal places is  $x = 2.7320508075689$ .)

To find the other roots, simply place the cursor to the immediate right of the = sign on the second row, i.e. immediately after  $X=$  and press  $\boxed{\text{CLEAR}}$  . Type a value for  $x$  which is **close** to the next root that you wish to find, say  $x = -1$ .

Press  $\boxed{\text{ALPHA}} \boxed{\text{ENTER}}$

While the calculator is busy 'thinking' you will see small bars scrolling at the top right of the screen.

Even though this method did not require you to actually graph the equation, it is usually convenient to quickly graph the equation so that you can find approximate values for the roots.

## For you to try

Solve the equation  $2x^3 - 6x^2 + 3x + 2 = 0$

Your answers should be  $x = -0.33602540378445$ ,  $1.3660254037843$ , and  $2$ .

These answers have used the maximum possible number of decimal places given by the calculator.

If you had any problems, see below for a full explanation of the keystrokes needed.

## Solution

For the equation  $2x^3 - 6x^2 + 3x + 2 = 0$

enter  $Y1 = 2x^3 - 6x^2 + 3x + 2$  and press **GRAPH** .

This is so that you can get an idea of where the roots will be.

*(It seems that there is one solution between  $-1$  and  $0$ , another just a bit more than  $1$  and the third seems to be very close to  $2$ .)*

Now press **CLEAR** to return to the Home screen.

Press **MATH** 0 *This enters the instruction **Equation solver***

Now enter the expression  $2x^3 - 6x^2 + 3x + 2$  in the conventional manner followed by **ENTER**

Indicate that you want a solution close to  $-1$  and then press **ALPHA**  
**ENTER**

The solution which is close to  $-1$  will then be calculated.

The root which is close to  $-1$  is  $-0.36602540378443$

To find the next solution,

press **CLEAR** anywhere to the right of  $X=$  and then enter a value which is close to the next solution.

Again press **ALPHA** **ENTER** to find the next solution.

The root which is close to  $1$  is  $1.366025404$  and the third solution is found to be exactly  $2$ .

This procedure has found that (to 4 d.p.)  $x = -0.3360$ ,  $1.3660$ , and  $2$

## Graph views

On the TI-83, there are different ways that you can view a graph.

### Example

Graph  $Y1 = -x^2 + 4x$  using a window of  $-2 \leq x \leq 6$ ,  $-2 \leq y \leq 6$  and both  $Xscl = Yscl = 1$ .

After pressing GRAPH you will see the graph with the standard view.

Press Y= then use the cursor keys to select the  $\backslash$  mark to the left of Y1.

Press ENTER





You will notice that the  $\backslash$  mark changes to a thick oblique line. This changes the graph to one with a thick line.

Press GRAPH to have a peek.

Go back to Y= and again select the mark to the left of Y1.

Successively pressing ENTER changes the mark to the following options. Try them, and then look at the resulting graph.

Here's what each option does.

$\backslash$	This is the usual (default) setting.
	This gives the graph with a thick line.
	This puts shading above the curve.
	Shading is now below the curve.
-o	A 'ball' traces the graph and leaves a path behind it.
o	The 'ball' traces the path, but leaves no permanent curve on the screen.
	The graphing line is dotted